SYNOPTIC: Two-Dimensional Potential Flow Theory for Multiple Bodies in Small Amplitude Motion, J. P. Giesing, *AIAA Journal*, Vol. 8, No. 11, pp. 1944–1953.

Airplane and Component Aerodynamics; Aircraft Gust Loading and Wind Shear; Unsteady Aerodynamics; Aeroelasticity and Hydroelasticity.

Theme

This paper describes a very general method for determining the unsteady, two-dimensional, incompressible flow about one or more bodies of arbitrary shape. Specifically the surface pressure, forces, and moments are obtained for bodies oscillating with small amplitude about their steady equilibrium position and on bodies in small amplitude gust fields. The equilibrium configuration of the bodies may also be arbitrary. The method described extends the oscillatory theory on several fronts; 1) a body of arbitrary shape is accommodated with ease, 2) bodies performing small oscillations about any equilibrium position are considered and 3) the present method uses a potential-flow procedure that does not require transformation methods, and therefore, allows for more than one body. As a result, the aerodynamic interference of several bodies vibrating independently may be determined, whether or not they possess circulation. Calculated results for single bodies in and out of gust fields are compared to results obtained by existing methods. Calculations and comparisons are also presented for two interacting bodies.

Content

The governing equations of unsteady potential-flow are the Laplace equation and the Bernoulli equation. In addition, a vorticity-conservation condition must be applied. Specifically, any loss of circulation of a body must show up in the wake of that body as shed vorticity so that the total vorticity of the system remains constant. The shed vortices flow off the body at the trailing edge and down the trailing streamline with the speed of the surrounding fluid. It is assumed that the location of the trailing streamline and the velocity along it are given to first order, by their steady values.

The unsteady boundary-value problem involving moving boundaries is reduced to an unsteady problem involving stationary boundaries. Fixing the boundaries for a single body is easily accomplished by simply adopting body-fixed coordinates. This is not true for more than one body since, generally, there will be relative motion among the bodies. Since the motions are assumed small, an expansion in the amplitude of motion about the steady position can be effected. Such an expansion gives an equivalent normal velocity boundary condition $\bar{w}_e$ on the stationary boundaries that involve both the steady w_s as well as the unsteady $\bar{w}_\delta$ motions

$$(\bar{w}_e) = \{(\bar{w}_\delta) + [\bar{C}_\delta](w_s)\}e^{j\omega t}$$

In a similar way there is an equivalent tangential flow, q_e, and potential ϕ_e that is dependent on the steady as well as the unsteady motion

$$(\bar{q}_e) = \{(\bar{q}_\delta) + [\bar{D}_\delta](w_s)\}e^{j\omega t}$$

$$(\bar{\phi}_e) = \{(\bar{\phi}_\delta) + [\bar{E}_\delta](w_s)\}e^{j\omega t}$$

The matrices $[\bar{C}_\delta]$, $[\bar{D}_\delta]$, and $[\bar{E}_\delta]$ represent the interaction between the unsteady motion of $0(\delta)$ and the steady flow of $0(1)$ and ω is the frequency of oscillation.

The solution of the Neumann boundary value problem for fixed boundaries is solved using a surface singularity distribution of the source type. The body surfaces are broken up into a series of straight line elements over which the source strength is assumed constant. The strengths of the elements are adjusted until the boundary conditions are satisfied. If A_{ij} and B_{ij} represents the normal and tangential velocity, respectively, at element midpoint i due to element j of unit strength, then the solution may be written symbolically as

$$\{\bar{q}_\delta\} = [B][A]^{-1}\{\bar{w}_e\}$$

The normal and tangential onset flow, $\bar{w}_\delta$ and $\bar{q}_\delta$ needed to form the equivalent normal and tangential onset flow $\bar{w}_e$ and $\bar{q}_e$ are determined from unsteady body motions, gusts fields and flowfields generated by the circulation within, and vortex-sheets shed from each body. The unsteady motions are made up of pitching, plunging, and horizontal oscillations. The gust fields considered are either moving or stationary. In addition a gust field generated by a water wave is considered. The flowfields generated by vortex sheet wakes are calculated using numerical integration. The vortex sheet is broken up into short straight line elements over which the steady fluid velocity is held constant. Over each element then the vorticity is sinusoidal and the flowfield due to the element is expressible in terms of exponential integrals.

Once the tangential velocity, $\bar{q}_e$, and potential, $\bar{\phi}_e$, are known the pressure C_p can be found.

$$C_p = -(2/V_\infty)(q_s\bar{q}_e + j\omega\bar{\phi}_e + \mathbf{V}_s\cdot\mathbf{V}_\delta)$$

where q_s is the steady tangential velocity and $\mathbf{V}_s$ and $\mathbf{V}_\delta$ are the steady and unsteady velocities of a surface point. Forces and moment are determined from the pressures.

An example of a two-body case is presented in the Fig. 1. The flap of a NACA 23012 airfoil is made to vibrate in rotation about an equilibrium incidence of 45°. The figure presents the configuration as well as the resulting pressure distribution. The complex lift coefficients for the main airfoil and flap are $-1.50 + i\,0.18$ and $-0.588 - i\,0.18$, respectively.

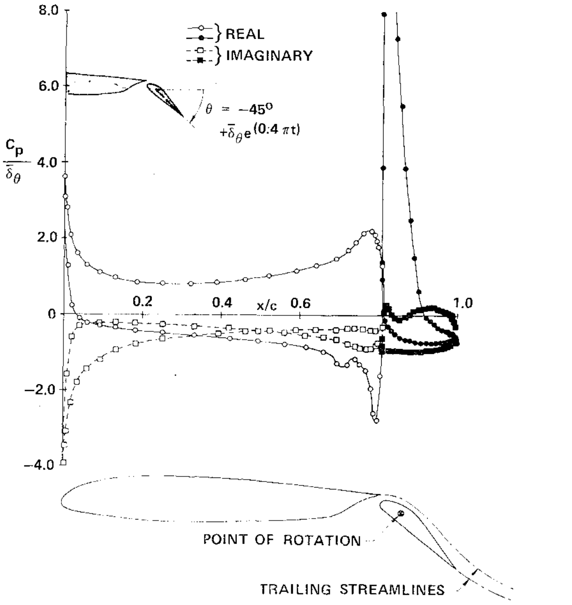

Fig. 1 Pressure distributions on a NACA 23012 airfoil and flap.